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CP and T Trajectory Diagrams for a Unified Graphical Representation of Neutrino Oscillations

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Abstract

Recently the CP trajectory diagram was introduced to demonstrate the difference between the intrinsic CP violating effects to those induced by matter for neutrino oscillation. In this paper we introduce the T trajectory diagram. In these diagrams the probability for a given oscillation process is plotted versus the probability for the CP- or the T-conjugate processes, which forms an ellipse as the CP or T violating phase is varied. Since the CP and the T conjugate processes are related by CPT symmetry, even in the presence of matter, these two trajectory diagrams are closely related with each other and form a unified description of neutrino oscillations in matter.

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I. INTRODUCTION

Accumulating evidences for neutrino oscillation in the atmospheric [1], the solar [2], and the accelerator [3] neutrino experiments make it realistic to think about exploring the full structure of the lepton flavor mixing. One of the challenging goals in such attempt would be to measure the leptonic CP or T violating phase δ in the MNS matrix [4]¹. It appears that the long baseline neutrino oscillation experiments are the most feasible way to actually detect these effects.

It has been known since sometime ago that the measurement of CP and T violating effects in the long baseline neutrino oscillation experiments can be either contaminated or enhanced by the matter effect inside the earth; see e.g., Refs. [5] and [6]. Therefore, it is one of the most important issues to achieve a complete understanding of the interplay between the CP-phase and the matter effects in parameter regions relevant for such experiments.

Toward the goal, we have introduced in a previous paper [7] the “CP trajectory diagram in bi-probability space” as a useful tool for pictorial representation of the CP and the matter effects in neutrino oscillation. This diagram enables us to display three effects; (a) genuine CP violation due to the $\sin \delta$ term, (b) CP conserving $\cos \delta$ term, and (c) fake CP violation due to the earth’s matter, separately in a single diagram.

In this paper, we introduce a related but an entirely new diagram, “T trajectory diagram in bi-probability space”. We demonstrate that the T trajectory diagram is by itself illuminative and is complementary with the CP diagram. More importantly, when combined with the CP diagram, it completes and unifies our understanding of the interplay between the effects due to CP or T-violating phase and the matter. The intimate relationship between the T and the CP diagrams reflects the underlying CPT theorem. Although the CPT theorem itself is broken by the presence of matter, this breaking can be compensated by a change in the sign of all the Δm^2 ’s.

¹In this paper we will assume that the light neutrino sector consists of only three active neutrinos.

II. CP AND T TRAJECTORY DIAGRAMS IN BI-PROBABILITY SPACE AND THEIR SYMMETRIES

Now we introduce the T trajectory diagram in bi-probability space spanned by $P(\nu) \equiv P(\nu_e \rightarrow \nu_\mu)$ and its T-conjugate $T[P(\nu)] \equiv P(\nu_\mu \rightarrow \nu_e)$. We fully explain in this section its notable characteristic properties, the relationship (or unity) with the CP trajectory diagram, and the symmetry relations obeyed by them.

Suppose that we compute the oscillation probability $P(\nu)$ and $T[P(\nu)]$ with a given set of oscillation and experimental parameters. Then, we draw a dot on the two-dimensional plane spanned by $P(\nu)$ and $T[P(\nu)]$. When δ is varied we have a set of dots which forms a closed trajectory, closed because the probability must be a periodic function of δ , a phase variable. Let us remind the reader that the T trajectory diagram has very similar structure with the CP trajectory diagram introduced in Ref. [7]; the abscissa is the same and the ordinate for the CP diagram is $CP[P(\nu)] \equiv P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)$.

In Fig. 1 the CP and the T trajectory diagrams, denoted as $CP\pm$ and $T\pm$, are plotted in the same figure. Here, the ordinate of the diagram is meant to be $CP[P(\nu)]$ for the CP and $T[P(\nu)]$ for the T diagrams, respectively. In the center there exist two vacuum diagrams, $V\pm$, one for each of the signs of Δm_{31}^2 . When the matter effect is turned on, the positive and the negative Δm_{31}^2 trajectories split.

In vacuum, the CP and the T trajectories are identical with each other. This is because the CPT theorem tells us that $T[P(\nu)] \equiv P(\nu_\mu \rightarrow \nu_e) = P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu) = CP[P(\nu)]$. Depending upon the sign of Δm_{31}^2 we have two CP (or T) trajectories which are almost degenerate with each other in vacuum due to the approximate symmetry under the simultaneous transformation [7]

$$\begin{aligned} \delta &\rightarrow \pi - \delta \pmod{2\pi}, \\ \Delta m_{31}^2 &\rightarrow -\Delta m_{31}^2. \end{aligned} \tag{1}$$

It was noticed that the CP and the T trajectories are elliptical exactly in vacuum and

approximately in matter [7]. Recently, it was shown in a remarkable paper by Kimura, Takamura, and Yokomakura [8] that it is exactly elliptic even in matter; the δ -dependence of the oscillation probabilities in constant matter density can be written on general ground in the form

$$\begin{aligned} P(\nu) &= A(a) \cos \delta + B(a) \sin \delta + C(a) \\ CP[P(\nu)] &= A(-a) \cos \delta - B(-a) \sin \delta + C(-a) \\ T[P(\nu)] &= A(a) \cos \delta - B(a) \sin \delta + C(a) \end{aligned} \tag{2}$$

where a denotes neutrino's index of refraction in matter, $a = \sqrt{2}G_F N_e$, with electron number density N_e and the Fermi constant G_F . The dependence on other variables are suppressed. Throughout this paper, we use the standard parameterization of the MNS matrix.

Matter effects split the two trajectories in quite different manners. Namely, the T trajectories split along the straight line $P(\nu) = T[P(\nu)]$ whereas the CP trajectories split in the orthogonal direction.² The T trajectories must move along the diagonal line because the $\delta = 0$, π points on T trajectories must stay on the diagonal because T violation vanishes at $\delta = 0$, π for matter distributions which are symmetric about the mid-point between production and detection. In constant matter density, this stems from the Naumov-Harrison-Scott relation (i.e., the proportionality between the vacuum and the matter Jarlskog factors) [9,10] and the absence of $\sin \delta$ dependence [11] in the other parts of the oscillation probability, or simply from the KTY formula in Eq. (2).

On the other hand, the CP trajectories must split along the direction orthogonal to the diagonal line for small matter effect. This is because the first order matter correction, which is proportional to $a = \sqrt{2}G_F N_e(x)$, has opposite sign for the neutrino and the anti-neutrino

² While it may be easier for the readers to understand the following discussions by referring the KTY formula in Eq. (2), our subsequent discussion will be entirely independent from the constant density approximation. We however rely on the adiabatic approximation when we utilize perturbative formulas.

oscillation probabilities; $\Delta P(\nu) = -\Delta P(\bar{\nu})$ [7]. No matter how large the matter effect the line connecting the two CP trajectories with opposite signs of Δm_{31}^2 is orthogonal to the diagonal line. As we will explain below this reflects a symmetry relationship obeyed by the oscillation probabilities, which we denote as the “CP-CP relation”.

One may also notice, as indicated by the eye-guided lines in Fig. 1 that the CP and the T trajectories have identical lengths when projected onto either the abscissa or the ordinate. It should be the case for the projection onto the abscissa because the abscissa is common for both of the CP and the T diagrams. What is nontrivial is the equality in length when projected onto the ordinate. It again represents a symmetry which we want to call the “T-CP relation”. We note that neither the CP-CP nor the T-CP relations are exact, as one may observe from Fig.1.

The precise statement of the CP-CP relation is (see Fig. 1)

$$\begin{aligned} P(\nu_e \rightarrow \nu_\mu; \Delta m_{31}^2, \Delta m_{21}^2, \delta, a) &= P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu; -\Delta m_{31}^2, -\Delta m_{21}^2, \delta, a) \\ &\approx P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu; -\Delta m_{31}^2, +\Delta m_{21}^2, \pi + \delta, a). \end{aligned} \quad (3)$$

Whereas the precise statement of the T-CP relation is

$$\begin{aligned} P(\nu_\mu \rightarrow \nu_e; \Delta m_{31}^2, \Delta m_{21}^2, \delta, a) &= P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu; -\Delta m_{31}^2, -\Delta m_{21}^2, 2\pi - \delta, a) \\ &\approx P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu; -\Delta m_{31}^2, +\Delta m_{21}^2, \pi - \delta, a). \end{aligned} \quad (4)$$

The equalities have the opposite sign for all the Δm^2 's from the left hand side to the right hand side. But because of the hierarchy, $|\Delta m_{31}^2| \gg |\Delta m_{21}^2|$, changing the sign of Δm_{21}^2 produces only a small deviation whose precise origin and magnitude will be become clear in the derivation of these relations. These relations imply that when the CP+ trajectory winds counter-clockwise as δ increases the CP- and T- (T+) trajectories wind clockwise (counter-clockwise) as in Fig.1.

To derive the CP-CP and T-CP relationships we need a number of identities. The first set of identities comes from taking the time reversal and the complex conjugate of

the neutrino evolution equation assuming that the matter profile is symmetric about the mid-point between production and detection;

$$P(\nu_\alpha \rightarrow \nu_\beta; \Delta m_{31}^2, \Delta m_{21}^2, \delta, a) = P(\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha; \Delta m_{31}^2, \Delta m_{21}^2, \delta, -a). \quad (5)$$

These identities are just CPT invariance in the presence of matter.

The second set of identities comes from taking the complex conjugate of the neutrino evolution equation for an arbitrary matter distribution, they are

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\beta; \Delta m_{31}^2, \Delta m_{21}^2, \delta, a) &= P(\nu_\alpha \rightarrow \nu_\beta; -\Delta m_{31}^2, -\Delta m_{21}^2, 2\pi - \delta, -a) \\ &= P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta; -\Delta m_{31}^2, -\Delta m_{21}^2, \delta, a) = P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta; \Delta m_{31}^2, \Delta m_{21}^2, 2\pi - \delta, -a) \end{aligned} \quad (6)$$

These identities relate the (anti-) neutrino oscillation probabilities in matter to those in anti-matter with opposite signs for all the Δm^2 's. They also relate the neutrino oscillation probabilities in matter to the anti-neutrino oscillation probabilities in matter with opposite signs for all the Δm^2 .

Combinations of these identities give the equalities in the CP-CP and the T-CP relations, the first lines in Eqs. (3) and (4). Now we will use the hierarchy that $|\Delta m_{31}^2| \gg |\Delta m_{21}^2|$ to get an approximate equality if we flip the sign of the Δm_{21}^2 with the appropriate change in the phase δ . The probability for $\nu_\alpha \rightarrow \nu_\beta$ consists of three terms as in Eq. (2). The coefficients A and B vanish like Δm_{21}^2 as $\Delta m_{21}^2 \rightarrow 0$. Thus, a change in the sign of Δm_{21}^2 in these two coefficients can be compensated by replacing δ with $\pi + \delta$, whereas the change in the coefficients C is further suppressed by an extra factor of $\sin \theta_{13}$. This proves the approximate equalities in the CP-CP and the T-CP relations.

The CP-CP relation guarantees that the size and the shape of two sign-conjugate ($\Delta m_{31}^2 = \pm |\Delta m_{31}^2|$) diagrams are identical, whereas there is no such relation in T diagrams. The CP phase relation between two sign-conjugate CP diagrams implies, among other things, that the two-fold ambiguity in δ which still remains after accurate determination of θ_{13} [7] are related by $\delta_2 = \delta_1 + \pi$ apart from the correction of order $\sim \frac{\Delta m_{21}^2}{\Delta m_{31}^2}$.

Finally, we note that from the first line of Eq. (6) that

$$P(\nu_\alpha \rightarrow \nu_\beta; \Delta m_{31}^2, \Delta m_{21}^2, \delta, a) \approx P(\nu_\alpha \rightarrow \nu_\beta; -\Delta m_{31}^2, \Delta m_{21}^2, \pi - \delta, -a) \quad (7)$$

which is a generalization of the approximate symmetry under the transformation (1) into the case in matter, from which the CP-CP and the T-CP relations also follow.

III. FURTHER EXAMPLES

While Fig. 1 is for the preferred parameters of a neutrino factory [12] it is interesting to see how the T-CP trajectory diagram changes as we change the energy and path length of the experiment. In Fig. 2 we have increased the path length to 6000 km and given the CP-T diagram for neutrino energies of 13 and 26 GeV. Again one can see how well the CP-CP and T-CP relationships hold.

The next examples use an energy which is approximately half the resonance energy corresponding to Δm_{31}^2 as suggested by Parke and Weiler [6]. At a distance of 3500 km this energy maximizes T-violation effects which is reflected in the size of ellipses. Halving the distance between source and detector for this energy produces a T-CP trajectory diagram which is similar to the neutrino factory diagram, Fig. 1 but with larger probabilities and asymmetries.

The final examples are using the energies and baselines of NUMI/MINOS and JHF/SK see Fig. 4. For NUMI/MINOS there is reasonable separation between the two Δm_{31}^2 ellipse whereas for JHF/SK there is some overlap. From the viewpoint of simultaneous determination of δ and the sign of Δm_{31}^2 the longer baseline of NUMI/MINOS would be more advantageous, while there are possibilities that it can be done at JHF/SK if P and $\text{CP}[P]$ are asymmetric [7,13],

Another feature which is worth noting in the JHF/SK case in Fig. 4 is that the problem of parameter degeneracy in T violation measurements is milder than that in CP measurements. It is because the T (or CP) ellipse is flatter in the radial direction, i.e., along the movement of T trajectory due to matter effect. The underlying reason for the phenomenon is that the coefficient of the $\cos \delta$ term oscillates and hence is always smaller than

the coefficient of the $\sin \delta$ term when averaged over energy width of a beam. In this sense, T measurement, if feasible experimentally, is more advantageous than CP measurement for simultaneous determination of δ and the sign of Δm_{31}^2 .

IV. PARAMETER DEGENERACY IN T-VIOLATION MEASUREMENTS

Armed by the T as well as the CP trajectory diagrams we are now ready to discuss the problem of parameter degeneracy with T-violation measurement. We do not aim at complete treatment of the problem in this paper but briefly note the new features that arise in T-violation measurement as compared to the CP-violation measurement. They arise because of the more symmetric nature of the T conjugate probability as apparent in Eq. (2).

We work in the same approximation as in Ref. [14] and write the oscillation probability in small s_{13} approximation. We have four equations:

$$\begin{aligned} P(\nu)_{\pm} &= X_{\pm}\theta^2 + Y_{\pm}\theta \cos\left(\delta \mp \frac{\Delta_{31}}{2}\right) + P_{\odot} \\ T[P(\nu)]_{\pm} &= X_{\pm}\theta^2 + Y_{\pm}\theta \cos\left(\delta \pm \frac{\Delta_{31}}{2}\right) + P_{\odot} \end{aligned} \quad (8)$$

where X_{\pm} and Y_{\pm} are given in Ref. [14], P_{\odot} indicates the term which is related with solar neutrino oscillations, and $\Delta_{31} \equiv \frac{|\Delta m_{31}^2|L}{2E}$. Note that \pm here refers to the sign of Δm_{31}^2 and θ is an abbreviation of $\theta_{13} \simeq s_{13}$.

It is easy to show that Eq. (8) can be solved to obtain the same-sign Δm_{31}^2 degenerate solutions as

$$\delta_2 = \pi - \delta_1 \quad \text{and} \quad \theta_2 - \theta_1 = \frac{Y_{\pm}}{X_{\pm}} \cos \delta_1 \cos\left(\frac{\Delta_{31}}{2}\right). \quad (9)$$

Notice that it is the degeneracy in matter though it looks like the one in vacuum [7].

Two $T\pm$ trajectories may overlap for a distance shorter than those in Fig.4, which would result in a mixed-sign degeneracy as in the case of CP measurement mentioned before. However, one can show that in the case of T violation measurement there is no more ambiguity in δ once θ_{13} is given;

$$\delta_2 = \delta_1 \quad \text{and} \quad \cos \delta_1 = -\theta_{13} \frac{(X_+ - X_-)}{2Y_+ \cos\left(\frac{\Delta_{31}}{2}\right)} \quad (10)$$

apart from the computable higher order correction due to the matter effect. This is obvious from Fig.4(b). A fuller treatment of the parameter degeneracy with T violation measurement will be reported elsewhere [15].

V. DISCUSSION AND CONCLUSIONS

First, it is worth pointing out that in matter the sensitivity to the CP or T-violating phase, δ , is larger for the T-violating pair ($\nu_\alpha \rightarrow \nu_\beta$, $\nu_\beta \rightarrow \nu_\alpha$) than for the CP-violating pair ($\nu_\alpha \rightarrow \nu_\beta$, $\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta$) of processes assuming $\Delta m_{31}^2 > 0$. For $\Delta m_{31}^2 < 0$ one should use the following T-violating pair, ($\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta$, $\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha$). This can be seen by comparing the size of the T ellipses to that of the CP ellipses in all of the previous diagrams. Unfortunately the experimental challenges associated with performing a T-violating experiment have yet to be overcome.

Due to the existence of the T-CP relationships for neutrino oscillations it is instructive and useful to add the T-violating trajectories to the CP-violating trajectories first proposed by Minakata and Nunokawa. The size and position of the T-violating ellipses can be easily estimated using simple arguments including the effects of matter and from them the CP-violating ellipses can be estimated. Thus, the combination of the CP and T trajectories form a unified picture of the CP-matter interplay in neutrino oscillations.

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REFERENCES

- [1] Kamiokande Collaboration, Y. Fukuda *et al.*, Phys. Lett. B **335** (1994) 237;
Super-Kamiokande Collaboration, Y. Fukuda *et al.*, Phys. Rev. Lett. **81** (1998) 1562;
S. Fukuda *et al.*, *ibid.* **85** (2000) 3999.
- [2] Homestake Collaboration, B. T. Cleveland *et al.*, Astrophys. J. **496** (1998) 505;
SAGE Collaboration, J. N. Abdurashitov *et al.*, Phys. Rev. C **60** (1999) 055801;
GALLEX Collaboration, W. Hampel *et al.*, Phys. Lett. B **447** (1999) 127;
GNO Collaboration, M. Altmann *et al.* Phys. Lett. B **490** (2000) 16;
Super-Kamiokande Collaboration, S. Fukuda *et al.*, Phys. Rev. Lett. **86** (2001) 5651;
ibid. **86** (2001) 5656;
SNO Collaboration, Q. R. Ahmad *et al.*, Phys. Rev. Lett. **87** (2001) 071301.
- [3] K2K Collaboration, S. H. Ahn *et al.*, Phys. Lett. B **511** (2001) 178;
See also <http://neutrino.kek.jp/news/2001.07.10.News/index-e.html>.
- [4] Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. **28** (1962) 870.
- [5] J. Arafune and J. Sato, Phys. Rev. D **55** (1997) 1653; J. Arafune, M. Koike and J. Sato,
Phys. Rev. D **56** (1997) 3093 [Erratum *ibid.* D **60** (1999) 119905];
H. Minakata and H. Nunokawa, Phys. Rev. D **57** (1998) 4403; Phys. Lett. B **413** (1997)
369; Phys. Lett. B **495** (2000) 369; Nucl. Instrum. Meth. A **472** (2001) 421;
K. Dick, M. Freund, M. Lindner and A. Romanino, Nucl. Phys. B **562** (1999) 29;
O. Yasuda, Acta. Phys. Polon. B **30** (1999) 3089;
M. Koike and J. Sato, Phys. Rev. D **61** (2000) 073012; Erratum *ibid.* D **62** (2000)
079903.
- [6] S. J. Parke and T. J. Weiler, Phys. Lett. B **501** (2001) 106.
- [7] H. Minakata and H. Nunokawa, JHEP **0110** (2001) 001 [hep-ph/0108085].
- [8] K. Kimura, A. Takamura and H. Yokomakura, hep-ph/0203099.

- [9] V. N. Naumov, Int. J. Mod. Phys. D **1** (1992) 379.
- [10] P. F. Harrison and W. G. Scott, Phys. Lett. B **476** (2000) 349.
- [11] H. W. Zaglauer and K. H. Schwarzer, Z. Phys. C **40** (1988) 273.
- [12] A. Cervera, A. Donini, M. B. Gavela, J. J. Gomez Cadenas, P. Hernandez, O. Mena and S. Rigolin, Nucl. Phys. B **579** (2000) 17 [*Erratum-ibid.* B **593** (2000) 731];
C. Albright *et al.*, hep-ex/0008064;
V. Barger, S. Geer, R. Raja and K. Whisnant, Phys. Rev. D **63** (2001) 113011;
J. Pinney and O. Yasuda, Phys. Rev. D **64** (2001) 093008.
- [13] H. Minakata and H. Nunokawa, Talk presented at 7th International Workshop on Topics in Astroparticle and Underground Physics (TAUP2001), Laboratori Nazionali del Gran Sasso, Italy, September 8-12, 2001, hep-ph/0111131, to appear in Nucl. Phys. Proc. Suppl.
- [14] J. Burguet-Castell, M.B. Gavela, J.J. Gomez-Cadenas, P. Hernandez and O. Mena, Nucl. Phys. B **608** (2001) 301.
- [15] H. Minakata, H. Nunokawa, and S. J. Parke, work in progress.

FIGURES

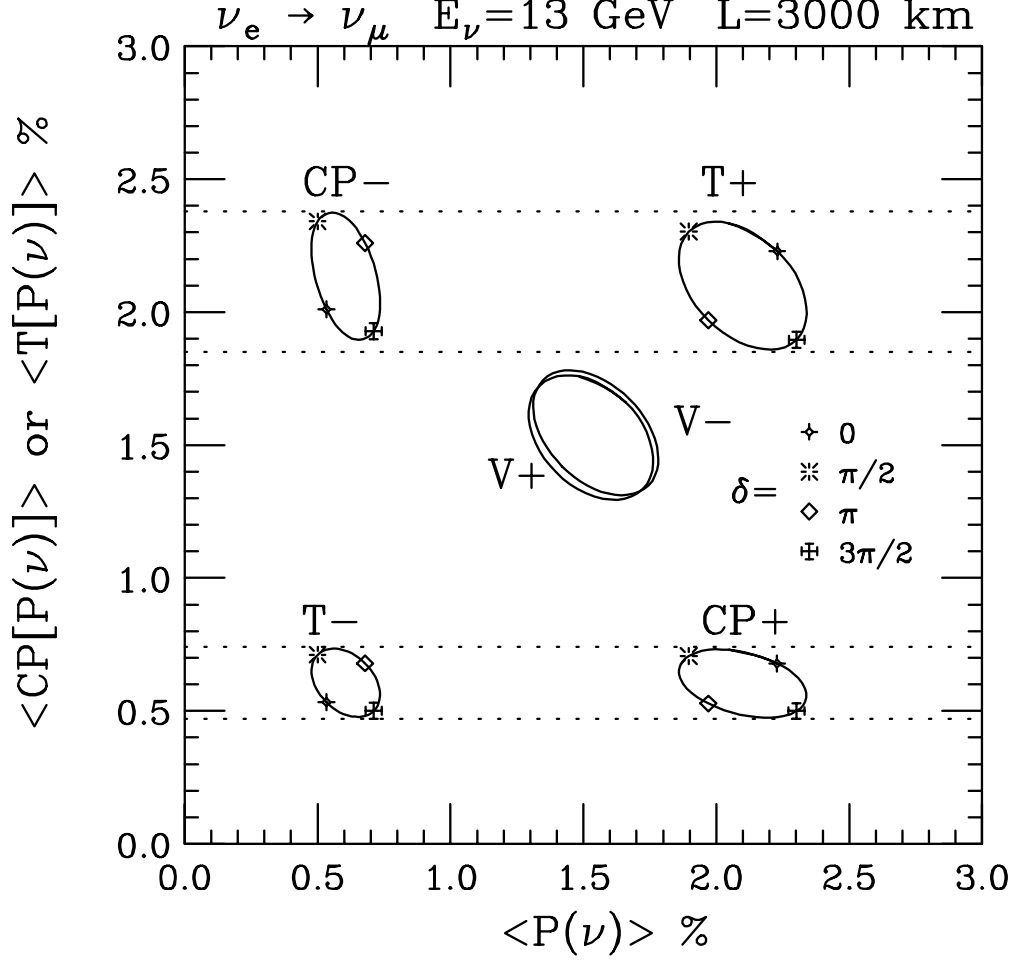


FIG. 1. The T (CP) trajectory diagrams (ellipses) in the plane $P(\nu_e \rightarrow \nu_\mu)$ verses $P(\nu_\mu \rightarrow \nu_e)$ ($P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)$) for an average neutrino energy of 13 GeV, spread 20%, and baseline of 3000 km. The ellipses labelled with a T and CP are in matter with a density times electron fraction given by $Y_e \rho = 1.5 \text{ g cm}^{-3}$ whereas those ellipses labelled V are in vacuum where the T and CP trajectories are identical. The plus or minus indicates the sign of Δm_{31}^2 . The mixing parameters are fixed to be $|\Delta m_{31}^2| = 3 \times 10^{-3} eV^2$, $\sin^2 2\theta_{23} = 1.0$, $\Delta m_{21}^2 = +5 \times 10^{-5} eV^2$, $\sin^2 2\theta_{12} = 0.8$ and $\sin^2 2\theta_{13} = 0.05$. The dotted lines are to guide the eye in demonstrating the T-CP relationship given by eqn(3). The marks on the CP and T ellipses are the points where the CP or T violating phase $\delta = (0, 1, 2, 3)\pi/2$ as indicated.

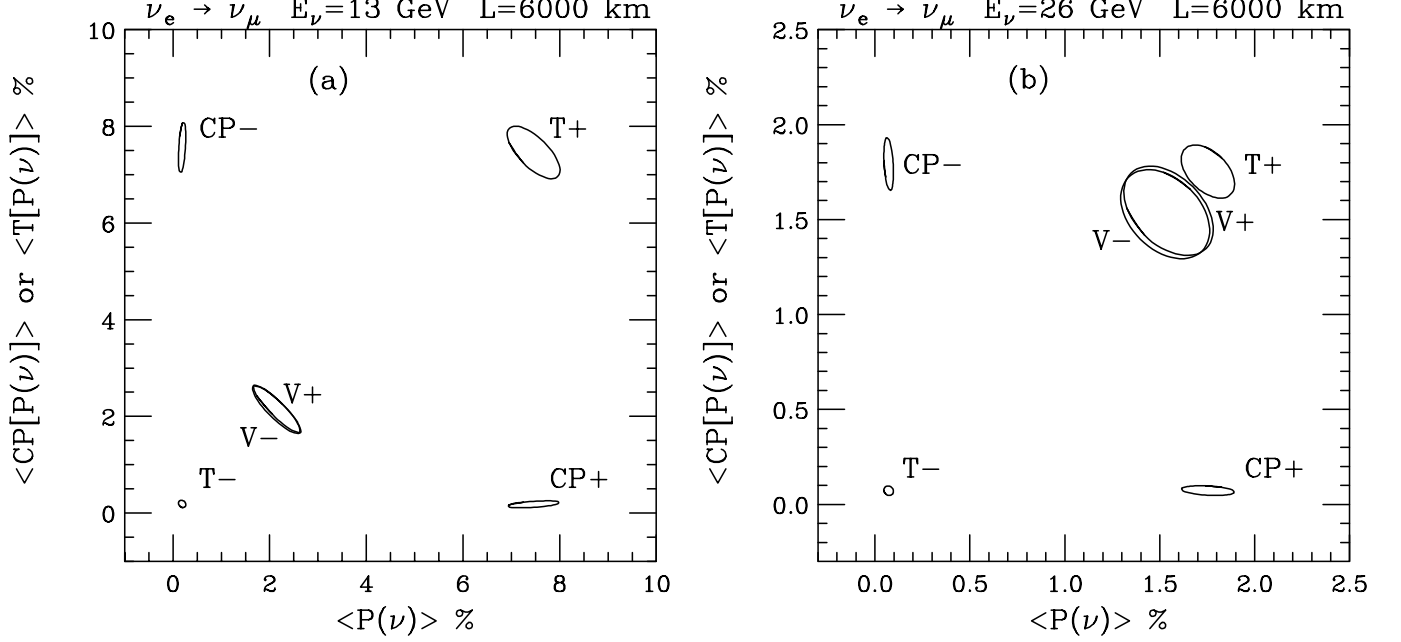


FIG. 2. The T (CP) trajectory diagrams (ellipses) in the plane $P(\nu_e \rightarrow \nu_\mu)$ versus $P(\nu_\mu \rightarrow \nu_e)$ ($P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)$) for an average neutrino energy (spread 20%) and baseline of (a) 13 GeV and 6000 km and (b) 26 GeV and 6000 km. Labels and mixing parameters are as in Fig. 1.

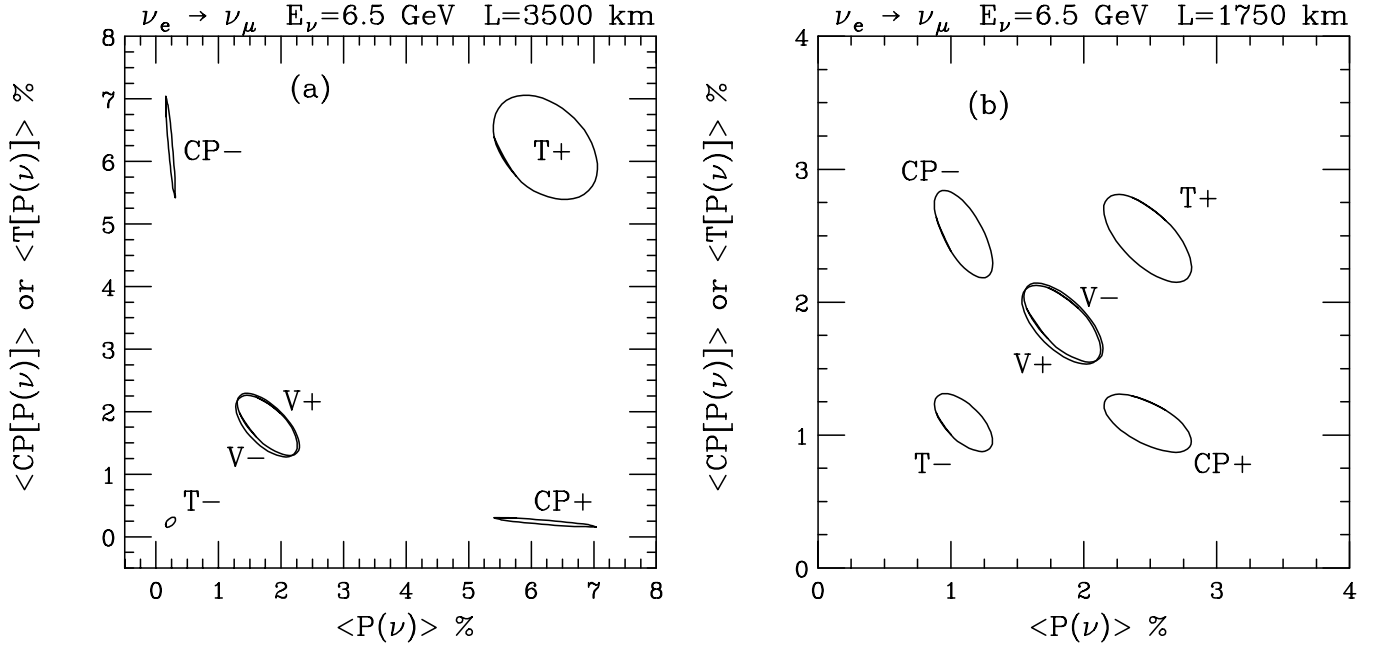


FIG. 3. The T (CP) trajectory diagrams (ellipses) in the plane $P(\nu_e \rightarrow \nu_\mu)$ versus $P(\nu_\mu \rightarrow \nu_e)$ ($P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)$) for an average neutrino energy (spread 20%) and baseline of (a) 6.5 GeV and 3500 km and (b) 6.5 GeV and 1750 km. Labels and mixing parameters are as in Fig. 1.

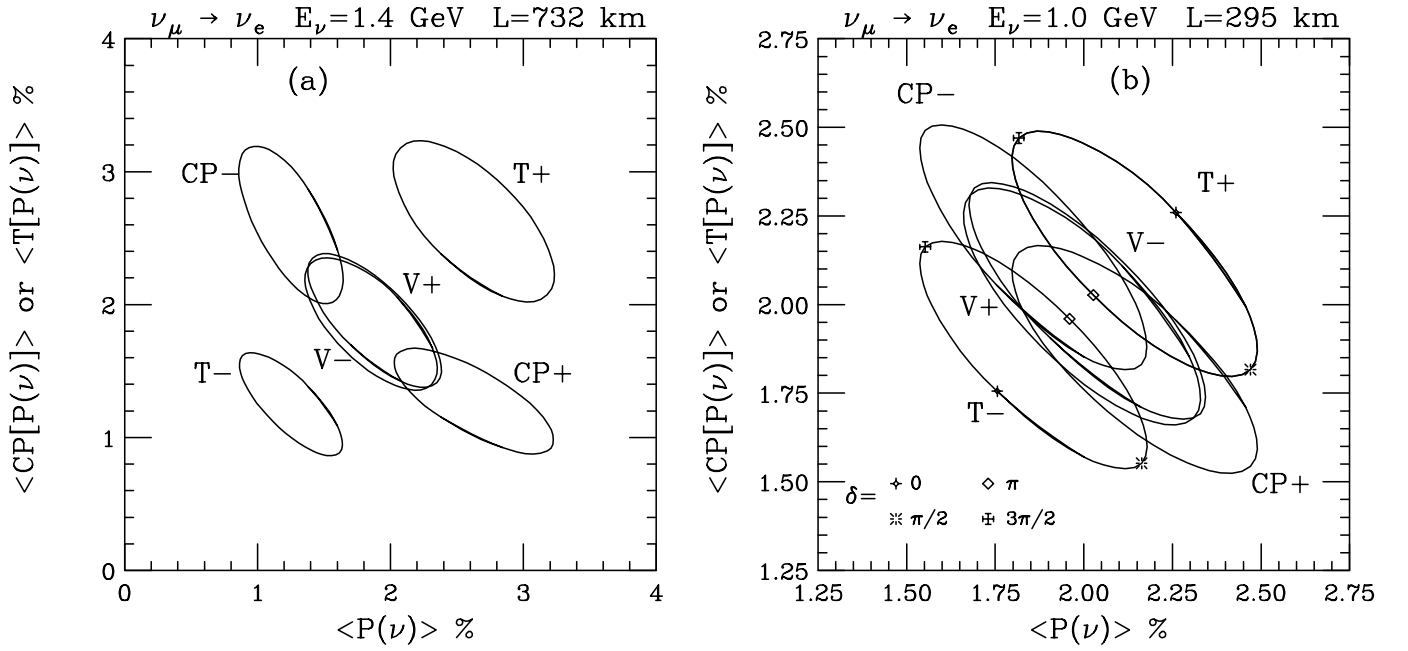


FIG. 4. The T (CP) trajectory diagrams (ellipses) in the plane $P(\nu_\mu \rightarrow \nu_e)$ versus $P(\nu_e \rightarrow \nu_\mu)$ ($P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$) for an average neutrino energy (spread 20%) and baseline of (a) 1.4 GeV and 732 km (NUMI/MINOS) and (b) 1.0 GeV and 295 km (JHF/SK). Labels and mixing parameters are as in Fig. 1.